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THE PERIODIC STRUCTURE OF AN ELECTRIC FIELD IN STRATIFIED PLASMA WITH TENSOR CONDUCTIVITY

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An exact solution of the problem of current distribution in a nonuniform plasma in a magnetic field between nonconducting planes is presented. Stratified plasma whose parameters vary in alternate cells is examined. In the here considered problem of a two-phase model of nonuniformities the electric field pattern is periodic and is defined by the interrelationship of local properties of plasma in cells, and also by the cell geometry. The determination of the electric field reduces to a step-by-step solution of two Riemann boundary problems of an even doubly-periodic function. The derived solution is used for calculating the effective conductivity and the Hall parameter, which makes it possible to establish, within the limits of the exactly solvable electrodynamic model, the anomalous properties of plasma in a strong magnetic field. Formulas for calculating a plane electric field in an anisotropically conducting medium in a rectangular region in the presence of two perfect electrodes are derived from the general solution as a particular case.

1. The periodic field pattern in a band. In conditions of conduction anisotropy induced by the Hall effect the current distribution is sensitive to the nonuniformity of parameters of the medium. In a strong magnetic field even small variation of the medium properties substantially affect the electric field pattern.

Let us consider the stratified nonuniformity of parameters of a medium, which may occur, for example, in magnetohydrodynamic installations in which the gas flows through a channel with alternate regions of high and low temperature or, when easily ionized substances are intermittently supplied to the gas. Such periodic nonuniformities of the moving plasma are sometimes deliberately induced in magnetohydrodynamic energy transformers of alternating currents.

It should be noted that stratification of the conductivity of a medium is a fairly common phenomenon. It is observed, for example, when an electric current passing through a gas leads to ionization instability; the discharge of a positively charge column is usually stratified in a wide range of gas pressure variation. Another example of stratified field pattern is provided by semiconductors with nonuniform distribution of impurities.

For the investigation of electric fields in such systems we use the following model of the problem. Let the medium in which the Hall effect is present be in a magnetic field between two nonconducting parallel planes, and let the parameters (conductivity and the Hall coefficient) of the medium have two different values in consecutive rectangular cells (Fig. 1).

Let us examine the two-dimensional current distribution with the use of the following system of equations:

$$\mathbf{j} + \frac{\beta}{H}\mathbf{j}\times\mathbf{H} - \sigma\mathbf{E} = 0, \qquad \nabla\cdot\mathbf{j} = 0, \qquad \nabla\times\mathbf{E} = 0 \tag{1.1}$$

where the generally accepted notation is used. We assume that in the band $0 < x < h$ the parameters $\mathbf{j}$, $\mathbf{E}$, $\mathbf{H}$, σ and β are defined by the following specified functions of coordinates:

$$\mathbf{j} = (j_x(x, y), j_y(x, y), 0), \quad \mathbf{E} = (E_x(x, y), E_y(x, y), 0), \quad \mathbf{H} = (0, 0, H_z(x))$$

$$\sigma, \beta, H = \begin{cases} \sigma_1, \beta_1, H_1 \quad \text{for } 0 < x < h,\ 2(k-1)l < y < (2k-1)l \\ \text{and } -2kl < -y < (1-2k)l \\ \sigma_2, \beta_2, H_2 \quad \text{for } 0 < x < h,\ (2k-1)l < y < 2kl \\ \text{and } (1-2k)l < -y < 2(1-k)l \end{cases}$$

Hence the problem admits periodic variation of the magnetic field. The heterogeneous properties of the medium may be caused by the variation of $\mathbf{H}$ or by some other factors. In the particular case the magnetic field is assumed to be uniform throughout the band. With these assumptions we can introduce in each cell the complex current

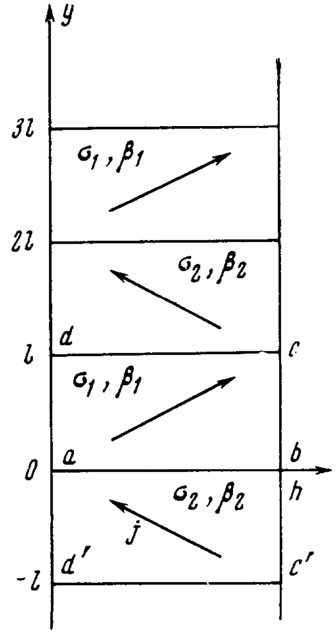

Fig. 1

$$j(z) = j_x(x, y) - ij_y(x, y)$$

where $(z = x + iy)$ To formulate boundary conditions for $j(z)$ it is sufficient to consider only two adjacent cells, since, owing to the problem symmetry, the field pattern repeats itself (Fig. 1). At the cell boundaries the following relationships must be satisfied:

$$j_{x1} = 0 \quad \text{along } da \text{ and } bc \tag{1.2}$$
$$j_{x2} = 0 \quad \text{along } ad' \text{ and } c'b$$
$$j_{y1} = j_{y2}, \qquad E_{x1} = E_{x2} \quad \text{along } ab,\ cd \text{ and } d'c'$$

where subscripts 1 and 2 denote parameters of two adjacent cells. The positive direction of following around the cell boundary is, as usual, that in which the cell region remains on the left. The first two expressions in (1.2) determine the condition of absence of current flow through the insulating planes and the remaining two follow from the general conditions at the boundary separating two heterogeneous media. The total current flowing along the band

$$I = \int_0^h j_y(x)\,dx \tag{1.3}$$

is assumed to be specified.

Let us pass to the solution of our problem. We denote the rectangle $abcd$ by S^+, and by S^- the rectangle symmetric to it about the real axis, and introduce two auxilliary piecewise-holomorphic functions $\Psi_1(z)$ and $\Psi_2(z)$

$$\Psi_1(z) = \begin{cases} \Psi_1^+(z) = j_1(z) = j_{x1}(x, y) - ij_{y1}(x, y), & z \in S^+ \\ \Psi_1^-(z) = \bar{j}_1(z) = j_{x1}(x, -y) + ij_{y1}(x, -y), & z \in S^- \end{cases}$$

$$\Psi_2(z) = \begin{cases} \Psi_2^+(z) = \bar{j}_2(z) = j_{x2}(x, -y) + ij_{y2}(x, -y), & z \in S^+ \\ \Psi_2^-(z) = j_2(z) = j_{x2}(x, y) - ij_{y2}(x, y), & z \in S^- \end{cases} \tag{1.4}$$

We assume that functions (1.4) are automorphic

$$\Psi_i(z) = \Psi_i(z') \qquad (i = 1, 2)$$

with respect to the group of substitutions

$$z' = z + 2mh + 2nil, \qquad z' = -z \tag{1.5}$$

throughout the whole plane of the complex variable. Each of the rectangles S^+ and S represents a fundamental region of group (1.5) whose basic function is the Weierstrass elliptic function $\wp(z)$. The latter can have any value in the fundamental region only once [1].

To investigate the stated problem of field we use the theory of the Riemann boundary value problem for automorphic functions, as proposed in [2, 3]. We denote by $\gamma(t)$ the normal component of current $j_y(t)$ at cell boundaries $L_1 = bc + da$, $L_2 = ad' + c'b$ and $L_3 = ab + cd + a'c'$. Using (1.4) and (1.5), from the first three formulas in (1.2) we obtain two nonuniform Riemann boundary value problems for four doubly-periodic functions

$$\begin{cases} \Psi_1^+(t) = -\Psi_1^-(t) & \text{along } L_1 \\ \Psi_1^+(t) = \Psi_1^-(t) - 2i\gamma(t) & \text{along } L_3 \end{cases}$$
$$\begin{cases} \Psi_2^+(t) = -\Psi_2^-(t) & \text{along } L_2 \\ \Psi_2^+(t) = \Psi_2^-(t) + 2i\gamma(t) & \text{along } L_3 \end{cases} \tag{1.6}$$
$$(t \in L = L_1 + L_2 + L_3)$$

We assume that $\gamma(t)$ satisfies Hölder's condition and consider solutions which outside the boundary line L have the property

$$\overline{\Psi}_i(\bar{z}) = \Psi_i(z) \qquad (i = 1, 2) \tag{1.7}$$

Problems of holomorphic functions in rectangles S^+ and S^-, which differ only by the sign of their free term, correspond to the Riemann problems (1.6). This implies the symmetry of the electric current pattern about the real axis. The solution of problems (1.6) (which are omitted for brevity) yields for currents along L_3 the relationships

$$j_{x1}(t) = -j_{x2}(t), \qquad t \in L_3 \tag{1.8}$$

Conditions (1.2) and (1.8) provide the complete system of boundary values for functions $j_i(z)$ $(i = 1, 2)$ in rectangles S^+ and S^-. It has to be borne in mind that $\gamma(t)$ along L_3 in (1.6) is not a priori known, hence it is necessary to establish in the final solution the admissibility of assumptions made with respect to function $\gamma(t)$. Using formulas (1.2) and (1.8) and Ohm's law (1.1), from (1.4) we obtain for $\Psi_1(z)$ the following homogeneous Riemann problem with discontinuous coefficients:

$$\Psi_1^+(t) = -\frac{\sigma_1 + \sigma_2 - i(\sigma_2\beta_1 - \sigma_1\beta_2)}{\sigma_1 + \sigma_2 + i(\sigma_2\beta_1 - \sigma_1\beta_2)}\Psi_1^-(t) \text{ along } L_3$$
$$\Psi_1^+(t) = -\Psi_1^-(t) \text{ along } L_1 \tag{1.9}$$

For function $\Psi_2(z)$ the Riemann problem is similar. The general solution of problem (1.9) which satisfies conditions (1.2) is

$$\Psi_1(z) = C\exp\left(-\frac{\pi i}{2} + \pi i\varepsilon\right)\left[\frac{\mathrm{cn}\,u}{\mathrm{sn}\,u\,\mathrm{dn}\,u}\right]^{2\varepsilon}$$
$$u = \left(\frac{Kz}{h}; k\right), \qquad \varepsilon = \frac{1}{\pi}\mathrm{arc\,tg}\,\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2} \tag{1.10}$$

where C is a constant, K is a complete elliptic integral of the first kind, k is the elliptic integral modulus, h is the band width, and sn, cn and dn are Jacobi's functions. The term $(\mathrm{cn}\,u\,/\,\mathrm{sn}\,u\,\mathrm{dn}\,u)^{2\varepsilon}$ is assumed to be the branch which is holomorphic in the rectangle S^+ and is positive along the boundary ab. In accordance with (1.4) function $\Psi_1(z)$ defined by formula (1.10) determines current distribution in S^+. Let us consider the behavior of the electric field along the rectangle boundary.

Along ab we have

$$\Psi_1^+(x) = C(\sin\pi\varepsilon - i\cos\pi\varepsilon)\,P(x)$$
$$P(x) = \left[\frac{\mathrm{cn}\,(Kx/h;\,k)}{\mathrm{sn}\,(Kx/h;\,k)\,\mathrm{dn}\,(Kx/h;\,k)}\right]^{2\varepsilon}, \qquad 0 < x < h \tag{1.11}$$

From (1.4) and Ohm's law we now have

$$j_{y1}(x) = CP(x)\left[1 + \left(\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\right)^2\right]^{-1/2} \tag{1.12}$$
$$j_{x1}(x) = \frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\,j_{y1}(x), \qquad \frac{\partial\varphi}{\partial x} = \frac{\beta_1 + \beta_2}{\sigma_1 + \sigma_2}\,j_{y1}(x)$$

Transforming elliptic functions by standard formulas [4], along bc we obtain

$$\Psi_1^+(y) = -iCR(y), \qquad R(y) = \left[\frac{\mathrm{sn}\,(Ky/h;\,k)\,\mathrm{dn}\,(Ky/h;\,k)}{\mathrm{cn}\,(Ky/h;\,k)}\right]^{2\varepsilon}$$
$$k' = \sqrt{1 - k^2}, \qquad 0 < y < l \tag{1.13}$$

and, consequently,

$$j_{y1}(y) = CR(y), \qquad \frac{\partial\varphi}{\partial y} = \frac{1}{\sigma_1}\,j_{y1}(y), \qquad j_{x1}(y) = 0 \tag{1.14}$$

Expressions for the electric field along cd are of the form (1.11) and (1.12), except that $P^{-1}(x)$ is to be substituted for $P(x)$. Similarly, in formulas for $j_{y1}(x)$ and $E_{y1}(y)$ along da it is necessary to substitute in (1.13) and (1.14) $R^{-1}(y)$ for $R(y)$. It will be seen that $j_y(x)$, i.e. $\gamma(x)$ satisfy along matching lines Hölder's condition, which is in agreement with the previously made assumptions about $\gamma(x)$ in the Riemann problems (1.6).

Constant C is determined by formula (1.3). Substituting into (1.3) $j_{y1}(x)$ from (1.11) and (1.12) and calculating the integral, we obtain

$$C = \frac{2IK(k)}{\pi hF\,[(1/2 + \varepsilon),\,(1/2 - \varepsilon);\,1;\,k^2]} \tag{1.15}$$

where F is a hypergeometric function. Formulas (1.4), (1.10) and (1.15) represent the complete solution of our problem.

The pattern of field in rectangle S^- is a mirror reflection of field S^+ symmetric about the real axis. This configuration is repeated in subsequent pairs of cells. The derived solution shows that the current distribution materially depends on the interrelationship of local parameters of the medium in adjacent cells.

Let us consider this problem in more detail. In the absence of the Hall effect ($\beta_1 = \beta_2 = 0$) the current in the band is uniform, i.e. $j_x = 0$ and $j_y = I/h$. This distribution also obtains in the presence of the Hall effect, provided throughout the band the

Hall coefficient R_H ($R_H = \beta / \sigma H$) remains constant in an everywhere homogeneous magnetic field $R_{H1} = R_{H2}$ ($H_1 = H_2$) or, when $H_1 \neq H_2$, that the more general condition $R_{H1}H_1 = R_{H2}H_2$ is satisfied. In all other cases the flow of electric current along the band is characterized by periodic bunching of streamlines at alternative boundaries of the band.

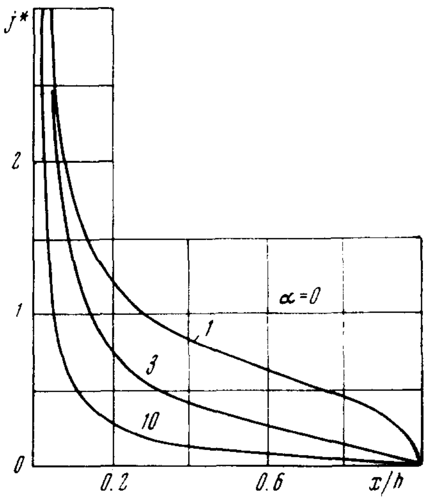

Fig. 2

The general pattern of the electric field may be considered to be the result of superposition of two particular distributions of current, viz, one, of the uniform longitudinal current j_y = const and the other of periodic current eddies generated by Hall's electromotive forces along the cell boundaries, where the potentiality of the electric field is disturbed.

The highest current concentration occurs along the contact lines of cells, where at one angle of the boundary the current infinitely increases (has an integrable singularity) and at the other vanishes altogether. These current distribution singularities at boundary angles alternate from one side of the band to the other.

There exists on the whole a kind of spatial modulation of the current of a period determined by the cell dimension. As an illustration, the curves of relative current $j^* = hj_y(x) / I$ are shown in Fig. 2 for four values of parameter $\alpha = (\sigma_1\beta_2 - \sigma_2\beta_1)/(\sigma_1 + \sigma_2) =$ 0, 1, 3 and 10. The curves are constructed with the use of formulas (1.12) and (1.15) and relate to square cells ($l / h = 1$). It will be seen from this diagram that the nonuniformity of current distribution in the cell increases with increasing parameter α.

2. Anomalous conductivity of medium in a strong magnetic field. As seen from the derived solution the electric field pattern in the presence of Hall effect in the plasma is determined by the heterogeneous properties of the medium. This reflects the general tendency of current distribution in a heterogeneous plasma, which, owing the fluctuation of its parameters, generates in the medium electric current eddies of the order of magnitude of the size of fluctuations. Such eddies result in distortion and entanglement of streamlines, the Joule dissipation increases, and the Hall field intensity diminishes. This phenomenon may be qualitatively linked with variation of the medium electric properties as a whole; it can be defined by the effective values of conductivity σ_{eff} and of the Hall parameter β_{eff}. The complex field pattern with local parameters σ and β of the medium is correlated to the "smoothed" field with effective parameters σ_{eff} and β_{eff}. With this approach it is possible to explain, within the limits of phenomenological theory, the physical aspects and the direction of the process in turbulent plasma by using Maxwell's equations and Ohm's law, without investigating the mechanism of plasma turbulence and resorting to equations of transport.

The problem of determination of the medium electric properties by a given fluctuation level is very general, since the phenomena leading for various reasons to its stratification can be included in the calculation scheme. The problem of determining σ_{eff} and β_{eff} was solved in [5] for the case of two-phase fluctuations in which plasma parameters had two discrete values in random distributed regions of equal area and considerably smaller dimensions than the complete system. The input equations (1.1) used here were also used for deriving the solution in [5]. If, however, the range of fluctuations is comparable to the characteristic dimension of the system, or they are orderly distributed in space, as they are in this case, the effective parameters must be determined by solving the boundary value problem.

Let us match an over-all uniform current distribution $j_x = 0$, and $j_y = I / h$ in a band containing plasma with constant parameters σ_{eff} and β_{eff} with the derived nonuniform current distribution. The condition of their equivalence is the equality in both cases of the total current flowing through the band, of the Hall emf, and of the voltage drop over the length $2l$ of the band.

In the first case these two voltages are defined by the formula

$$U_H = \frac{\beta_{eff}}{\sigma_{eff}} I, \qquad U = \frac{2}{\sigma_{eff}} \frac{l}{h} I \tag{2.1}$$

For a nonuniform current distribution the Hall emf along a contact line of adjacent cells and the voltage drop over the length of two cells, in accordance with (1.12) and (1.4), respectively, are defined by formulas

$$U_H = C \frac{\beta_1 + \beta_2}{\sigma_1 + \sigma_2} \left[1 + \left(\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\right)^2\right]^{-1/2} \int_0^h P(x)\,dx \tag{2.2}$$

$$U = C\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right) \int_0^l R(y)\,dy$$

Integrating (2.2) and substituting in the derived formulas the constant defined by (1.15), we obtain

$$U_H = I\,\frac{\beta_1 + \beta_2}{\sigma_1 + \sigma_2}, \qquad U = I\delta\,\frac{\sigma_1 + \sigma_2}{\sigma_1\sigma_2}\left[1 + \left(\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\right)^2\right]^{-1/2} \tag{2.3}$$

$$\delta = lF\left[\left(\frac{1}{2} + \varepsilon\right), \left(\frac{1}{2} - \varepsilon\right); 1; k'^2\right] \Big/ hF\left[\left(\frac{1}{2} + \varepsilon\right), \left(\frac{1}{2} - \varepsilon\right); 1; k^2\right]$$

where F is a hypergeometric function, and k and $k' = \sqrt{1 - k^2}$ are the moduli of elliptic integrals of the first kind. Equating the expressions for U_H and U defined, respectively, by (2.1) and (2.2), we obtain two expressions from which follow formulas

$$\sigma_{eff} = \frac{2\delta\sigma_1\sigma_2}{\sigma_1 + \sigma_2}\left[1 + \left(\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\right)^2\right]^{-1/2} \tag{2.4}$$

$$\beta_{eff} = \frac{2\delta\sigma_1\sigma_2(\beta_1 + \beta_2)}{(\sigma_1 + \sigma_2)^2}\left[1 + \left(\frac{\sigma_1\beta_2 - \sigma_2\beta_1}{\sigma_1 + \sigma_2}\right)^2\right]^{-1/2}$$

The derived formulas determine the effective parameters of the medium for $l / h \ll 1$. This condition must be satisfied, since the Hall emf varies along the band, when the medium parameters are variable. If this condition is satisfied, such variations can be neglected, otherwise it is necessary to average the Hall emf with respect to length. We would, furthermore, point out that for $l/h \gtrsim 1$ the introduction of effective parameters in the considered model has no purpose, since all characteristic dimensions of the nonuniformities are not small in comparison with the characteristic dimension (h) of the band.

Let us apply the derived formulas to specific cases.

1) If condition $\beta_1 / \sigma_1 = \beta_2 / \sigma_2$ is satisfied, formulas(2.4) reduce to

$$\sigma_{eff} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}, \quad \beta_{eff} = \frac{2\sigma_1\sigma_2(\beta_1 + \beta_2)}{(\sigma_1 + \sigma_2)^2} \quad (\delta = 1) \tag{2.5}$$

In this case the current distribution in the band is uniform ($j_x = 0$ and $j_y = \text{const}$). In the elementary theory of plasma the condition $\beta_1 / \sigma_1 = \beta_2 / \sigma_2$ is equivalent to the equality $H_1 / n_1 = H_2 / n_2$, when $\sigma = ne^2\tau/m$ and $\beta = e\tau H / mc$ (n, e, τ and m are the concentration, the charge, the collision time, and the mass of electrons, respectively, and c is the speed of light), and, if the magnetic field is uniform ($H_1 = H_2$), the fulfilment of this condition indicates the equality of electron concentration in adjacent cells. Velikhov's suggestion that it is possible to suppress ionization instability in low temperature plasma by complete ionization of the additive is based on this property.

2) If $\beta = \beta_1 = \beta_2$ and the plasma heterogeneity is related to variation of conductivity ($\sigma_1 \neq \sigma_2$), then for considerable values of the dimensionless parameter $\beta^2\Delta^2 \gg 1$ ($\Delta = |\sigma_1 - \sigma_2| / (\sigma_1 + \sigma_2)$ is the relative fluctuation of conductivity) we have

$$\sigma_{eff} = \frac{2l\sigma_1\sigma_2}{h\beta|\sigma_1 - \sigma_2|}, \qquad \beta_{eff} = \frac{2l\sigma_1\sigma_2}{h|\sigma_1^2 - \sigma_2^2|} \tag{2.6}$$

In the derivation of (2.6) from (2.4) allowance was made for $\delta \sim l / h$, when $\beta^2\Delta^2 \gg 1$. Formula (2.6) shows that with increasing magnetic field intensity the medium electric conductivity diminishes and the Hall parameter becomes saturated. Even a small nonuniformity of conductivity at high Hall parameters results in appreciable variation of properties of the medium as a whole. This defines the anomalous properties of plasma in a strong magnetic field. In the model considered here the conductivity diminishes in inverse proportion to the magnetic field intensity, while the Hall parameter is independent of the latter. It is interesting to note that a similar form of dependence on σ_{eff} and β_{eff} in a model of plasma with random distribution of nonuniformities is established in [5]. A distinctive feature of the derived result is that σ_{eff} and β_{eff} depend on linear dimensions of cells and that for $\sigma_2 / \sigma_1 \to 0$ β_{eff} behaves as $4l\sigma_2 / h\sigma_1$, i.e. it becomes saturated at the level determined by the ratio σ_2 / σ_1, as distinct from the results in [5], where $\beta_{eff} \to 1$, when $\sigma_2 / \sigma_1 \to 0$.

3) Another limit case in which $\sigma = \sigma_1 = \sigma_2$ and $\beta_1 \neq \beta_2$ can be similarly investigated. If the fluctuations are small $|\beta_1 - \beta_2| \ll 2$, then $\sigma_{eff} \to \sigma$ and $\beta_{eff} \to \beta_1 \sim \beta_2$, while for $|\beta_1 - \beta_2| \gg 2$ we have

$$\sigma_{eff} = \frac{2l\sigma}{h|\beta_1 - \beta_2|}, \qquad \beta_{eff} = \frac{l(\beta_1 + \beta_2)}{h|\beta_1 - \beta_2|} \tag{2.7}$$

This shows that the asymptotic formulas which define the properties of plasma in a strong magnetic field are the same, independently of whether it is only the Hall parameter or only the conductivity that fluctuates.

For analyzing the behavior of plasma in moderate magnetic fields, or when both local parameters σ and β vary simultaneously, it is necessary to use the general formulas (2.4).

3. Effective plasma parameters in the presence of ion slip.

In a strong magnetic field the Larmor rotation in low temperature plasma is imparted not only to electrons, but also to ions, hence Ohm's law is of the form

$$\mathbf{j}+\frac{\beta_e}{H}\,\mathbf{j}\times\mathbf{H}-\frac{\beta_e\beta_i}{H^2}(\mathbf{j}\times\mathbf{H})\times\mathbf{H}-\sigma\mathbf{E}=0$$

which is different from (1.1). Taking into account the ion slip does not require a new mathematical formulation of the problem. All necessary formulas are obtained from the previously derived expressions by substituting in the latter σ_ν^* and β_ν^* ($\nu = 1, 2$) for σ_ν and β_ν defined by

$$\sigma_\nu^*=\frac{\sigma_\nu}{1+\gamma_\nu\beta_\nu^2},\qquad \beta_\nu^*=\frac{\beta_\nu}{1+\gamma_\nu\beta_\nu^2},\qquad \gamma_\nu=\frac{\beta_{e\nu}}{\beta_{i\nu}}$$

Asymptotic formulas for the effective parameters of plasma in a strong magnetic field in which there is a considerable ion slip ($\gamma_\nu\beta_\nu^2\gg 1$) are in all cases of the same form

$$\sigma_{eff}=\frac{2\sigma_1\sigma_2}{\gamma_1\beta_1^2\sigma_2+\gamma_2\beta_2^2\sigma_1},\qquad \beta_{eff}=\frac{2\sigma_1\sigma_2\beta_1\beta_2(\gamma_1\beta_1+\gamma_2\beta_2)}{(\gamma_1\beta_1^2\sigma_2+\gamma_2\beta_2^2\sigma_1)^2}$$

The formulas for the plasma effective characteristics are now independent of linear dimensions of cells and the current distribution in the band is uniform. The effective Hall parameter is no longer saturated, as in (2) and (3) above, but deceases with increasing magnetic field intensity. On the whole, function $\beta_{eff}(H)$ is characterized by the presence of a maximum. In weak magnetic fields β_{eff} increases proportionally to H, while in strong magnetic fields, in which ion slip occurs, β_{eff} decreases in inverse proportion to H.

4. The solution derived in Sect. 1 is also applicable to the particular case of two-dimensional current distribution in a rectangular region with electrodes, which is relevant to the calculation of the field of a semiconductor plate with two symmetrically located electrodes in a uniform magnetic field. In this case the solution is obtained by passing to limit $\sigma_2\to\infty$. It differs from (1.10) and (1.15) only by the expression for the Hall angle $\pi\varepsilon$, instead of (1.10) we have

$$\varepsilon=\frac{1}{\pi}\operatorname{arc\,tg}(-\beta),\qquad 0\leqslant|\varepsilon|<\frac{1}{2}\qquad(\beta\equiv\beta_1)\tag{4.1}$$

The voltage drop between the electrodes is calculated by formula

$$U=\frac{C}{\sigma_1}\int_0^l R(y)\,dy\tag{4.2}$$

where C and $R(y)$ are defined by formulas (1.15) and (1.11) (for $\sigma_2\to\infty$). Using (4.2), we obtain for the plate total resistance the expression

$$\Omega=\frac{U}{I}=\frac{\sqrt{1+\beta^2}}{\sigma d}\,\frac{F\left[\left(\frac{1}{2}+\varepsilon\right),\left(\frac{1}{2}-\varepsilon\right);\,1;\,k'^2\right]}{F\left[\left(\frac{1}{2}+\varepsilon\right),\left(\frac{1}{2}-\varepsilon\right);\,1;\,k^2\right]}\tag{4.3}$$

where d is the plate thickness. Formula (4.3) reduces to the simple form $\Omega=\sqrt{1+\beta^2}/\sigma d$, when the plate is square ($l/h=1$).

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EXPLOSION IN A VARIABLE-DENSITY MEDIUM IN THE PRESENCE OF VARIABLE COUNTERPRESSURE

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The solution of the non-self-similar problem of explosion in a medium with variable initial density whose distribution is subject to power law is considered with variable initial pressure taken into consideration. An exact analytical solution is obtained in particular cases for the initial phase of explosion. The dependence of dimensionless parameters of motion on the geometric coordinate and the shock wave radius, which is obtained by solving differential equations, is derived in the solution of the complete non-self-similar problem. Derived solutions are used for calculating cases of spherical and cylindrical symmetry of explosion for various values of the determining parameters.

The one-dimensional self-similar problem of a strong point explosion was formulated and solved by Sedov [1, 2] on the assumption that the initial pressure of gas, which is small in comparison with the pressure at the front, can be neglected and that the initial density is constant. Strong explosion in a medium of varying density dependent on the geometric coordinate according to the power law was considered in [1, 3]. When counterpressure is taken into consideration, the problem becomes non-self-similar. Its numerical solution appeared in several publications [4 – 9], in which initial pressure was assumed constant.

The non-self-similar problem of explosion in a medium of varying initial density ρ_1 and varying initial pressure p_1 is considered here. These parameters are defined by

$$\rho_1 = Ar^{-\omega}, \qquad p_1 = Cr^{-\varkappa} \tag{0.1}$$

If $\varkappa = 2\omega - 2$, then, in the presence of a gravitational field, the initial density and pressure distributions (0.1) satisfy the equilibrium equations of the medium [1]. A particular case of this problem in linearized formulation for $\varkappa = \omega$ was investigated in [12, 13].

Considerable calculation difficulties encountered in non-self-similar problems have led to the appearance of several approximate methods [3–7, 11]. Sedov had suggested to construct approximate solutions of problems of unsteady motion